REPASO EXAMEN 2ª EV.

1.- $A = \begin{pmatrix} 3 & m \\ 1-m & m+1 \end{pmatrix}$

Hacemos el determinante y lo igualamos a 0

$$\begin{vmatrix} 3 & m \\ 1-m & m+1 \end{vmatrix} = 3(m+1) - m(1-m) = 3m+3-m+m^2 =$$

$$= m^2+2m+3 = 0$$

$$m = \frac{-2 \pm \sqrt{4-4\cdot1\cdot3}}{2} = \cancel{\frac{2 \pm \sqrt{-8}}{2}}$$

SOL = Para todo valor de m, la matriz tiene inversa

2.- $A = \begin{pmatrix} 1 & 0 & 2 \\ -2 & 1 & 0 \end{pmatrix}$ $B = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$

$$B \cdot B^t - A \cdot A^t = \begin{pmatrix} -2 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} -2 & 5 \end{pmatrix} - \begin{pmatrix} 1 & 0 & 2 \\ -2 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & -2 \\ 0 & 1 \\ 2 & 0 \end{pmatrix} =$$

$$= \begin{pmatrix} 4 & -10 \\ -10 & 25 \end{pmatrix} - \begin{pmatrix} 5 & -2 \\ -2 & 5 \end{pmatrix} = \boxed{\begin{pmatrix} -1 & -8 \\ -8 & 20 \end{pmatrix}}$$

3.- $A = \begin{pmatrix} 0 & 2 \\ 3 & 0 \end{pmatrix}$ $B = \begin{pmatrix} a & b \\ 6 & 1 \end{pmatrix}$

a) $A \cdot B = B \cdot A$

$$\begin{pmatrix} 0 & 2 \\ 3 & 0 \end{pmatrix} \cdot \begin{pmatrix} a & b \\ 6 & 1 \end{pmatrix} = \begin{pmatrix} a & b \\ 6 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 & 2 \\ 3 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 12 & 2 \\ 3a & 3b \end{pmatrix} = \begin{pmatrix} 3b & 2a \\ 3 & 12 \end{pmatrix} \Rightarrow \begin{array}{l} 12 = 3b \Rightarrow \boxed{b = 4} \\ 2 = 2a \Rightarrow \boxed{a = 1} \end{array}$$

b)

↓

b) $A = \begin{pmatrix} 0 & 2 \\ 3 & 0 \end{pmatrix}$ $B = \begin{pmatrix} 1 & 0 \\ 6 & 1 \end{pmatrix}$

$$X \cdot B - A = I$$
$$X \cdot B = I + A$$
$$X \cdot B \cdot B^{-1} = (I + A) \cdot B^{-1}$$
$$\boxed{X = (I + A) \cdot B^{-1}}$$ Siempre hay que ponerlo

$|B| = \begin{vmatrix} 1 & 0 \\ 6 & 1 \end{vmatrix} = 1$ $\quad Adj(B) = \begin{pmatrix} 1 & -6 \\ 0 & 1 \end{pmatrix}$

$Adj(B)^t = \begin{pmatrix} 1 & 0 \\ -6 & 1 \end{pmatrix}$

$$B^{-1} = \frac{1}{1} \cdot \begin{pmatrix} 1 & 0 \\ -6 & 1 \end{pmatrix} = \boxed{\begin{pmatrix} 1 & 0 \\ -6 & 1 \end{pmatrix}}$$

4.- x = € café
y = € cortado
z = café con leche

$$\left.\begin{aligned} 2x + 2y + z &= 3 \\ x + y + 3z &= 3'25 \\ x + 2y + z &= 2'45 \end{aligned}\right\}$$

$$\left(\begin{array}{ccc|c} 2 & 2 & 1 & 3 \\ 1 & 1 & 3 & 3'25 \\ 1 & 2 & 1 & 2'45 \end{array}\right) \qquad \begin{vmatrix} 2 & 2 & 1 \\ 1 & 1 & 3 \\ 1 & 2 & 1 \end{vmatrix} = 2+6+2-1-2-12 = -5$$

$$x = \frac{\begin{vmatrix} 3 & 2 & 1 \\ 3'25 & 1 & 3 \\ 2'45 & 2 & 1 \end{vmatrix}}{-5} = \frac{3 + 14'7 + 6'5 - 2'45 - 6'5 - 18}{-5} = \frac{-27'5}{-5} = \boxed{0'55\text{ €}}$$

$$y = \frac{\begin{vmatrix} 2 & 3 & 1 \\ 1 & 3'25 & 3 \\ 1 & 2'45 & 1 \end{vmatrix}}{-5} = \frac{6'5 + 9 - 2'45 - 3'25 - 3 - 14'7}{-5} = \frac{-3}{-5} = \boxed{0'6}\text{ €}$$

$$z = \frac{\begin{vmatrix} 2 & 2 & 3 \\ 1 & 1 & 3'25 \\ 1 & 2 & 2'45 \end{vmatrix}}{-5} = \frac{4'9 + 6'5 + 6 - 3 - 4'9 - 13}{-5} = \frac{-3'5}{-5} = \boxed{0'7}\text{ €}$$

5.- $x = €$ invierte en A

$y = €$ invierte en B

$z = €$ invierte en C

$$\left.\begin{array}{l} x+y+z = 12.000 \\ x = 2(y+z) \\ 0'04x + 0'05y - 0'02z = 432'5 \end{array}\right\}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 12000 \\ 1 & -2 & -2 & 0 \\ 4 & 5 & -2 & 43250 \end{array}\right)$$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -2 & -2 \\ 4 & 5 & -2 \end{vmatrix} = 4 - 8 + 5 + 8 + 2 + 10 = 21$$

$$x = \frac{\begin{vmatrix} 12000 & 1 & 1 \\ 0 & -2 & -2 \\ 43250 & 5 & -2 \end{vmatrix}}{21} = \frac{48000 - 86500 + 86500 + 120000}{21} = \boxed{8000}$$

$$y = \frac{\begin{vmatrix} 1 & 12000 & 1 \\ 1 & 0 & -2 \\ 4 & 43250 & -2 \end{vmatrix}}{21} = \frac{-96000 + 43250 + 24000 + 86500}{21} = \boxed{2750}$$

$$z = \frac{\begin{vmatrix} 1 & 1 & 12000 \\ 1 & -2 & 0 \\ 4 & 5 & 43250 \end{vmatrix}}{21} = \frac{-86500 + 60000 + 96000 - 43250}{21} = \boxed{1250}$$

6.- $$\left.\begin{array}{l} 3x + 2y \geq 0 \\ x - 2y \geq -1 \\ 5x + 4y \leq 16 \\ x - y \leq 5 \end{array}\right\}$$

$y = \frac{-3x}{2}$ 1

x	y
0	0
4	-6

$y = \frac{x+1}{2}$ 2

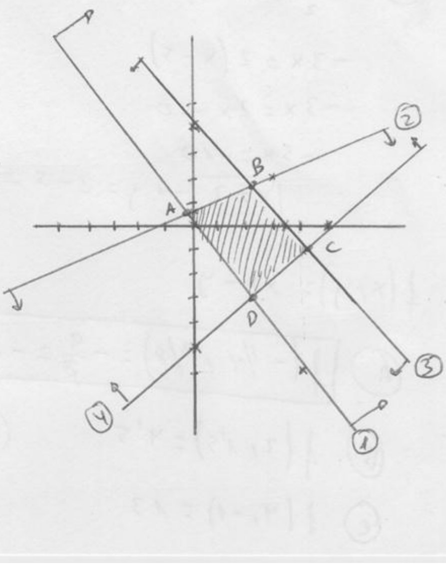
x	y
0	0'5
3	2

$y = \frac{16-5x}{4}$ 3

x	y
0	4
3'2	0

$y = x - 5$ 4

x	y
0	-5
5	0



(A) $\dfrac{-3x}{2} = \dfrac{x+1}{2}$

$-3x = x+1$

$-4x = 1$

$x = -\dfrac{1}{4} \longrightarrow y = -\dfrac{3}{2}\cdot\left(-\dfrac{1}{4}\right) = \dfrac{3}{8} \Rightarrow \boxed{A\left(-1/4,\ 3/8\right)}$

(B) $\dfrac{x+1}{2} = \dfrac{16-5x}{4}$

$4(x+1) = 2(16-5x)$

$4x+4 = 32-10x$

$14x = 28$

$\boxed{x=2} \longrightarrow y = \dfrac{2+1}{2} = \dfrac{3}{2} = 1'5 \Rightarrow \boxed{B\left(2,\ 1'5\right)}$

(C) $\dfrac{16-5x}{4} = x-5$

$16-5x = 4(x-5)$

$16-5x = 4x-20$

$-9x = -36$

$\boxed{x=4} \longrightarrow y = 4-5 = -1 \Rightarrow \boxed{C\left(4,\ -1\right)}$

(D) $-\dfrac{3x}{2} = x-5$

$-3x = 2(x-5)$

$-3x = 2x-10$

$-5x = -10$

$\boxed{x=2} \longrightarrow y = 2-5 = -3 \Rightarrow \boxed{D\left(2,\ -3\right)}$

$f(x,y) = 3x - y$

(A) $\boxed{f\left(-1/4,\ 3/8\right) = -\dfrac{9}{8} = -1'125}$ MÍNIMO

(B) $f(2,\ 1'5) = 4'5$

(C) $f(4,-1) = 13$

(D) $f(2,-3) = 9$

③

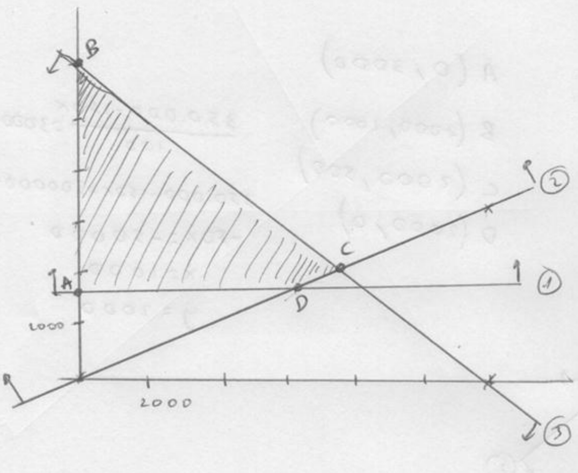
7.- x = € invierte en A $f(x,y) = 0'1x + 0'05y$

y = € invierte en B

$$\left.\begin{array}{l} y \geq 3000 \\ x \leq 2y \\ x+y \leq 12000 \\ x \geq 0 \\ y \geq 0 \end{array}\right\} \begin{array}{l} y = 3000 \ ① \\ y = \frac{x}{2} \longrightarrow ② \\ y = 12000 - x \longrightarrow ③ \end{array}$$

x	y
0	0
12000	6000

x	y
0	12000
12000	0



A (0,3000)

B (0,12000)

C $\frac{x}{2} = 12000 - x$

$x = 24000 - 2x$

$3x = 24000$

$x = 8000$

$y = 4000$

C (8000, 4000)

D (6000,3000)

$f(0,3000) = 150$

$f(0,12000) = 600$

$\boxed{f(8000,4000) = 1000}$ MÁXIMO.

$f(6000,3000) = 750$

Debe invertir 8000 € en A y 4000 € en B para obtener 1000 € de benef.

(8) x = microondas caro $f(x,y) = 15x + 11y$

y = " barato

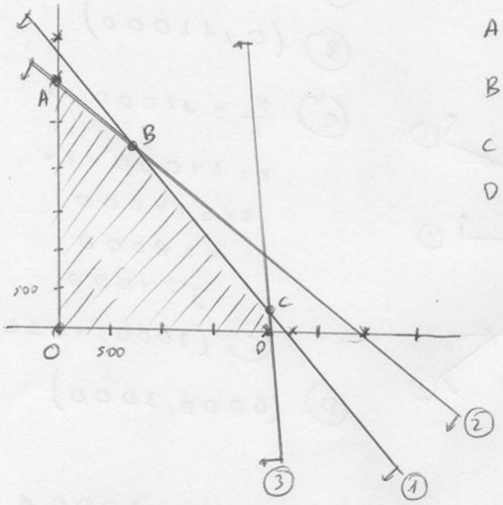
$$\left.\begin{array}{l} 150x + 100y \le 350000 \\ x + y \le 3000 \\ x \le 2000 \\ x \geq 0 \\ y \geq 0 \end{array}\right\}$$

$y = \dfrac{350.000 - 150x}{100}$ → (1)

x	y
0	3500
2333'3	0

$y = 3000 - x$ → (2)

x	y
0	3000
3000	0



A (0, 3000)

B (1000, 2000)

C (2000, 500)

D (2000, 0)

$\dfrac{350.000 - 150x}{100} = 3000 - x$

$350.000 - 150x = 300000 - 100x$

$-50x = -50000$

$x = 1000$

$y = 2000$

$f(0, 3000) = 33.000$

$\boxed{f(1000, 2000) = 37000}$ MÁXIMO

$f(2000, 500) = 35500$

$f(2000, 0) = 30.000$

Ha de comprar 1000 microondas caro y 2000 baratos para conseguir 37000 € de beneficio.

9.- $f(x) = \begin{cases} x+2 & \text{si } 0 \le x < 2 \\ x^2 - 6x + 12 & \text{si } 2 \le x \le 4 \\ -2x + a & \text{si } 4 < x \le 8 \end{cases}$

a) $\boxed{x = 2}$

$$\left.\begin{aligned} & f(2) = 4 \\ & \lim_{x \to 2^-} f(x) = 4 \\ & \lim_{x \to 2^+} f(x) = 4 - 12 + 12 = 4 \end{aligned}\right\} \text{Continua en } x = 2$$

$\boxed{x = 4}$

$$\left.\begin{aligned} & f(4) = 16 - 24 + 12 = 4 \\ & \lim_{x \to 4^-} f(x) = 4 \\ & \lim_{x \to 4^+} f(x) = -8 + a \end{aligned}\right\} \begin{aligned} & 4 = -8 + a \\ & \boxed{a = 12} \end{aligned}$$

b) $f'(x) = \begin{cases} 1 & \text{si } 0 < x < 2 \\ 2x - 6 & \text{si } 2 < x < 4 \\ -2 & \text{si } 4 < x < 8 \end{cases} \longrightarrow 2x - 6 = 0 \quad x = 3$

$f'(x)$	0 – 2	2 – 3	3 – 4	4 – 8
signo	+	−	+	−
$f(x)$	↗	↘	↗	↘

$f(0) = 2$

$\boxed{f(2) = 4}$

$f(3) = 9 - 18 + 12 = 3$

$\boxed{f(4) = 4}$

Máximos en $(2, 4)$
$(4, 4)$

(10) $f(x) = x^3 + rx^2 + sx + t \longrightarrow f'(x) = 3x^2 + 2rx + s$

Min en $x = -2 \longrightarrow f'(-2) = 0$

$f'(-2) = \boxed{12 - 4r + s = 0}$

Mini en $x = 0 \longrightarrow f'(0) = 0$

$f'(0) = \boxed{s = 0}$

Pasa $(1, -1) \longrightarrow f(1) = -1$

$f(1) = \boxed{1 + r + s + t = -1}$

$-4r + s = -12$

$-4r = -12$

$\boxed{r = 3}$

$r + s + t = -2$

$3 + t = -2$

$\boxed{t = -5}$

$\boxed{f(x) = x^3 + 3x^2 - 5}$

(12) Recta tg pasa por $(0,0) \longrightarrow y - 0 = m(x - 0)$

$y = mx$

$\boxed{y = -5x}$

$m = f'(0) = -5$

$f'(x) = 6x - 5$

(13) $f(x) = \begin{cases} 3x^2 + 1 & \text{si } x < 0 \\ x^3 + ax^2 & \text{si } 0 \leq x < 1 \\ bx + \ln x & \text{si } x \geq 1 \end{cases}$

Continuidad

$\boxed{x = 0}$

$\left.\begin{array}{l} f(0) = 1 \\ \lim_{x \to 0^-} f(x) = 1 \\ \lim_{x \to 0^+} f(x) = 0 \end{array}\right\}$ Discontinua inevitable de salto finito en $x = 0$ $\Longrightarrow$ No es derivable en $x = 0$ por no ser continua.

↓

$\boxed{x = 1}$

$f(1) = 1 + a$

$\lim\limits_{x \to 1^-} f(x) = 1 + a$

$\lim\limits_{x \to 1^+} f(x) = b + \ln 1 = b$

$\left.\right\} \boxed{b = 1 + a}$

Derivabilidad en $x = 1$

$$f'(x) = \begin{cases} 6x & \text{si } x < 0 \\ 3x^2 + 2a & \text{si } 0 < x < 1 \\ b + \frac{1}{x} & \text{si } x > 1 \end{cases}$$

$f'(1^-) = 3 + 2a$

$f'(1^+) = b + 1$

$\left.\right\} \boxed{3 + 2a = b + 1}$

$-a + \cancel{b} = 1$

$2a - \cancel{b} = -2$

$\boxed{a = -1} \longrightarrow b = 1 - 1 \longrightarrow \boxed{b = 0}$

(14)

$$p(t) = \begin{cases} 50 - t^2 & \text{si } 0 \leq t \leq 3 \\ 56 - \frac{20t}{t+1} & \text{si } t > 3 \end{cases}$$

a) $p(3) = 41$

$\lim\limits_{t \to 3^-} p(t) = 41$

$\lim\limits_{t \to 3^+} p(t) = 41$

$\left.\right\}$ Si es continua

c) $\lim\limits_{t \to +\infty} P(t) = 56 - \frac{20 \cdot \infty}{\infty + 1} = 56 - 20 = \boxed{36 \text{ Tm}}$ Si estoy de acuerdo

Por comparación de ∞

b) Comprobemos si es decreciente en todo su dominio

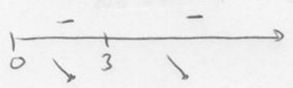
$$p'(t) = \begin{cases} -2t \\ -\frac{20 \cdot (t+1) - 20t}{(t+1)^2} = -\frac{20}{(t+1)^2} \end{cases}$$

$-2t = 0$

$\boxed{t = 0}$

$$\frac{-20}{(t+1)^2} = 0$$

$-20 \neq 0$ No hay



La función siempre es decreciente, por lo tanto sí es cierto que conforme pase el tiempo cada vez aguantará menos.

15.- a) $f(x) = \dfrac{x^2+1}{x^2-9}$

- Dom(f): $\mathbb{R} - \{\pm 3\}$

- Ptos de corte: $OX \to y = 0 \to \dfrac{x^2+1}{x^2-9} = 0$

$x^2 + 1 = 0 \to \cancel{x^2 = -1}$

$OY \to x = 0 \Rightarrow y = -\frac{1}{9} \Rightarrow \boxed{(0, -1/9)}$

- Asíntotas

$\boxed{\text{A. Horizontales}}$ en $\boxed{y = -1/9}$

$$y = \lim_{x \to \pm\infty} \frac{x^2+1}{x^2-9} = \boxed{1}$$

Comportamiento

$$\lim_{x \to \pm\infty} \frac{x^2+1}{x^2-9} - 1 = \lim_{x \to \pm\infty} \frac{\cancel{x^2}+1 - \cancel{x^2}+9}{(x^2-9)} = \lim_{x \to \pm\infty} \frac{10}{x^2-9} = 0$$

$0 \xrightarrow{+\infty} 0^+$ Por encima

$0 \xrightarrow{-\infty} 0^+$ Por encima

$\boxed{\text{A. Verticales}}$ $\boxed{x = 3}$ $\boxed{x = -3}$

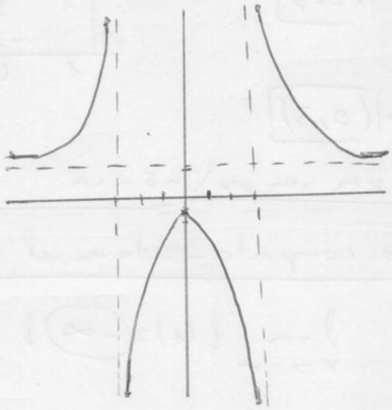
$$\lim_{x \to 3} \frac{x^2+1}{x^2-9} = \boxed{\frac{10}{0}} = \begin{cases} \lim_{x \to 3^-} f(x) = \dfrac{+}{-} = -\infty \downarrow \\ \lim_{x \to 3^+} f(x) = \dfrac{+}{+} = +\infty \uparrow \end{cases}$$

6

$$\lim_{x\to -3} \frac{x^2+1}{x^2-9} = \frac{10}{0} = \begin{cases} \lim_{x\to -3^-} f(x) = \frac{+}{+} = +\infty \uparrow \\ \lim_{x\to -3^+} f(x) = \frac{+}{-} = -\infty \downarrow \end{cases}$$

A. Oblicua

No hay por haber horizontales



Monotonía

$$f'(x) = \frac{2x(x^2-9)-(x^2+1)\cdot 2x}{(x^2-9)^2} = \frac{\cancel{2x^3}-18x-\cancel{2x^3}-2x}{(x^2-9)^2} = \frac{-20x}{(x^2-9)^2} = 0$$

$$-20x = 0 \Rightarrow x=0$$

$f'(x)$: $+$ | -3 | $+$ | 0 | $-$ | 3 | $-$

↗ ↗ ↘ ↘

Máximo en $(0, -1/9)$

$\uparrow$
$f(0)$

b) $f(x) = x^3 + x^2 - 5x + 3$

- $Dom(f) = \mathbb{R}$

- Ptos de corte

$OX \to y = 0 \Rightarrow x^3 + x^2 - 5x + 3 = 0$

	1	1	-5	3
1		1	2	-3
	1	2	-3	0
1		1	3	
	1	3	0	
-3		-3		
	1	0		

$\boxed{x = 1}$ $\boxed{x = -3}$

$\boxed{(1,0)\ (-3,0)}$

$OY \to x = 0 \Rightarrow y = 3 \Rightarrow \boxed{(0,3)}$

- Asíntotas $\to$ No hay por ser polinómica

- Rama parabólica o comportamiento en el ∞

$\lim_{x \to +\infty} f(x) = +\infty \uparrow$ $\quad \lim_{x \to -\infty} f(x) = -\infty \downarrow$

- Monotonía

$f'(x) = 3x^2 + 2x - 5 = 0 \longrightarrow$

	3	2	-5
1		3	5
	3	5	0

$x = 1$

$\to 3x + 5 = 0$

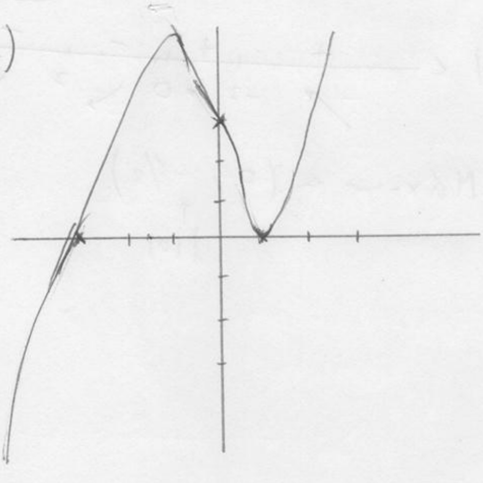
$x = -\frac{5}{3}$

$f'(x)$: $+$ $\quad -5/3 \quad$ $-$ $\quad 1 \quad$ $+$

$\nearrow \quad \searrow \quad \nearrow$

Máximo en $(-1'67, 9'48)$

Mínimo en $(1, 0)$



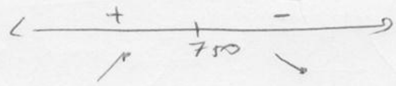
16 $I(x) = 28x^2 + 36.000x$

$G(x) = 44x^2 + 12000x + 700.000$

a) $B(x) = I(x) - G(x) = \boxed{-16x^2 + 24000x - 700.000}$

b) $B'(x) = -32x + 24000 = 0$

$\boxed{x = 750}$



c) $\boxed{\text{Máximo en } (750,\ 8.300.000)}$

17 $A = \begin{vmatrix} K & 1 & -1 \\ 0 & 2 & K \\ 4 & 0 & -K \end{vmatrix} = -2K^2 + 4K + 8 = 0$

$\boxed{K = \begin{cases} -1'225 \\ 3'225 \end{cases}}$

Si $K = -1'225$ o $K = 3'225 \longrightarrow$ No tiene inversa

Si $K \neq -1'225$ y $K \neq 3'225 \rightarrow$ Tiene inversa