REPASO TOTAL 2º BACH

1.- a) $3B - AX = I$

$-AX = I - 3B$

$AX = 3B - I$

$A^{-1} \cdot A \cdot X = A^{-1}(3B - I)$

$X = A^{-1}(3B - I)$

$|A| = \begin{vmatrix} 3 & -1 \\ 4 & 2 \end{vmatrix} = 6 + 4 = 10$

$Adj(A) = \begin{pmatrix} 2 & -4 \\ +1 & 3 \end{pmatrix}$

$A^{-1} = \frac{1}{10}\begin{pmatrix} 2 & 1 \\ -4 & 3 \end{pmatrix} = \begin{pmatrix} \frac{2}{10} & \frac{1}{10} \\ -\frac{4}{10} & \frac{3}{10} \end{pmatrix}$

$X = \begin{pmatrix} \frac{2}{10} & \frac{1}{10} \\ \frac{-4}{10} & \frac{3}{10} \end{pmatrix} \cdot \left[\begin{pmatrix} 12 & 0 \\ 3 & 6 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right] = \frac{1}{10}\begin{pmatrix} 2 & 1 \\ -4 & 3 \end{pmatrix} \cdot \begin{pmatrix} 11 & 0 \\ 3 & 5 \end{pmatrix} = \frac{1}{10}\begin{pmatrix} 25 & 5 \\ -35 & 15 \end{pmatrix}$

$\boxed{X = \begin{pmatrix} \frac{5}{2} & \frac{1}{2} \\ -\frac{7}{2} & \frac{3}{2} \end{pmatrix}}$

b) $A^t \cdot C + 2X = B$

$2X = B - A^t \cdot C$

$\frac{1}{2} \cdot 2 \cdot X = \frac{1}{2}\left[B - A^t \cdot C\right]$

$X = \frac{1}{2}\left[B - A^t \cdot C\right]$

$A^t = \begin{pmatrix} 3 & 4 \\ -1 & 2 \end{pmatrix}$

$X = \frac{1}{2}\left[\begin{pmatrix} 4 & 0 \\ 1 & 2 \end{pmatrix} - \begin{pmatrix} 3 & 4 \\ -1 & 2 \end{pmatrix} \cdot \begin{pmatrix} 2 & -1 & 1 \\ 1 & 0 & 2 \end{pmatrix}\right] =$

$X = \frac{1}{2}\left[\begin{pmatrix} 4 & 0 \\ 1 & 2 \end{pmatrix} - \begin{pmatrix} 10 & -3 & 11 \\ 0 & 1 & 3 \end{pmatrix}\right] =$

$X = \frac{1}{2}\left[\begin{pmatrix} 4 & 0 \\ 1 & 2 \end{pmatrix} + \right.$ No se puede, tienen diferente dimensión

c) $\left.\begin{array}{l} 2X - Y = A \\ X + 3Y = B \end{array}\right\}$ $\begin{array}{l} 2X - Y = A \\ -2X - 6Y = -2B \end{array}$

$-7Y = A - 2B$

$Y = -\frac{1}{7}(A - 2B)$

$A - 2B = \begin{pmatrix} 3 & -1 \\ 4 & 2 \end{pmatrix} - \begin{pmatrix} 8 & 0 \\ 2 & 4 \end{pmatrix} =$

$= \boxed{\begin{pmatrix} -5 & -1 \\ 2 & -2 \end{pmatrix}}$

$\boxed{Y = \begin{pmatrix} 5/7 & 1/7 \\ -2/7 & +2/7 \end{pmatrix}}$

$X = B - 3Y$

$X = \begin{pmatrix} 4 & 0 \\ 1 & 2 \end{pmatrix} - \begin{pmatrix} 15/7 & 3/7 \\ -6/7 & 6/7 \end{pmatrix}$

$\boxed{X = \begin{pmatrix} 13/7 & -3/7 \\ 13/7 & 8/7 \end{pmatrix}}$

2.- a) $rg(A) : \begin{pmatrix} 3 & 1 & 1 \\ -1 & 3 & -3 \\ 1 & 2 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 2 & -1 \\ -1 & 3 & -3 \\ 3 & 1 & 1 \end{pmatrix}^{(-3)} \sim \begin{pmatrix} 1 & 2 & 1 \\ 0 & 5 & -4 \\ 0 & -5 & 4 \end{pmatrix} \sim$

$\sim \begin{pmatrix} 1 & 2 & 1 \\ 0 & 5 & -4 \\ 0 & 0 & 0 \end{pmatrix} \Longrightarrow \boxed{rg(A) = 2} \Longrightarrow$ Por Gauss

b) $\boxed{|A| = 0}$, ya que $rg(A) = 2 <$ nº incógnitas $= 3$

3.- Para que $\exists A^{-1} \Rightarrow |A| \neq 0$

$$|A| = \begin{vmatrix} 2m & 5 & 1 \\ 1 & 1 & 1 \\ -2 & m & -2 \end{vmatrix} = -4m - \cancel{10} + m + 2 + \cancel{10} - 2m^2 = -2m^2 - 3m + 2 = 0$$

$$m = \begin{cases} -2 \\ 1/2 \end{cases}$$

Si $m \neq -2$ y $m \neq \frac{1}{2} \Rightarrow \exists A^{-1} \Rightarrow$ Es inversible

$$|A| = \begin{vmatrix} 4 & 5 & 1 \\ 1 & 1 & 1 \\ -2 & 2 & -2 \end{vmatrix} = -8 - \cancel{10} + 2 + 2 + \cancel{10} - 8 = -16 + 4 = -12$$

$A^{-1} = \begin{pmatrix} 1/3 & -1 & -1/[illegible] \\ 0 & 1/2 & 1/[illegible] \\ -1/3 & 3/2 & 1/[illegible] \end{pmatrix}$

4.- $\begin{pmatrix} 2 & -4 \\ 0 & 3 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 2 & 1 \\ -2 & 2 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \\ 2 \end{pmatrix}$

$$\left.\begin{aligned} 2x - 4y + 2y + z &= 4 \\ 3y - 2y + 2z &= 3 \\ 3x + \cancel{y} - \cancel{y} + z &= 2 \end{aligned}\right\} \quad \left.\begin{aligned} 2x - 2y + z &= 4 \\ y + 2z &= 3 \\ 3x \quad + z &= 2 \end{aligned}\right\} \quad \left(\begin{array}{ccc|c} 2 & -2 & 1 & 4 \\ 0 & 1 & 2 & 3 \\ 3 & 0 & 1 & 2 \end{array}\right)$$

Por Cramer

$$|A| = \begin{vmatrix} 2 & -2 & 1 \\ 0 & 1 & 2 \\ 3 & 0 & 1 \end{vmatrix} = 2 - 12 - 3 = \boxed{-13}$$

$$x = \frac{\begin{vmatrix} 4 & -2 & 1 \\ 3 & 1 & 2 \\ 2 & 0 & 1 \end{vmatrix}}{-13} = \frac{4 - 8 - 2 + 6}{-13} = \boxed{0}$$

$$y = \frac{\begin{vmatrix} 2 & 4 & 1 \\ 0 & 3 & 2 \\ 3 & 2 & 1 \end{vmatrix}}{-13} = \frac{6 + 24 - 9 - 8}{-13} = \frac{13}{-13} = \boxed{-1}$$

$$z = \frac{\begin{vmatrix} 2 & -2 & 4 \\ 0 & 1 & 3 \\ 3 & 0 & 2 \end{vmatrix}}{-13} = \frac{4 - 18 - 12}{-13} = \frac{-26}{-13} = \boxed{2}$$

5.- x = alumnos en A
y = " " B
z = " " C

$$\left.\begin{array}{l} x+y+z=60 \\ z-10=2x \\ y+10=x+z-10 \end{array}\right\} \quad \left\{\begin{array}{l} x+y+z=60 \\ -2x+z=10 \\ -x+y-z=-20 \end{array}\right.$$

Por Gauss

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 60 \\ -2 & 0 & 1 & 10 \\ -1 & 1 & -1 & -20 \end{array}\right)^{(1)} \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 60 \\ 0 & 2 & 3 & 130 \\ 0 & 2 & 0 & 40 \end{array}\right)$$

$\longrightarrow 2y+3z=130 \downarrow 40+3z=130$; $z=\frac{90}{3}$; $\boxed{z=30}$

$\longrightarrow 2y=40$; $\boxed{y=20}$

$x+y+z=60$
$x=60-20-30$
$\boxed{x=10}$

6.- x = cantidad invertida en A
y = " " " B
z = " " " C

$$\left.\begin{array}{l} x+y+z=27.000 \\ y=x+0'5x \\ \frac{1}{2}(x+z)=y \end{array}\right\} \quad \begin{array}{l} x+y+z=27.000 \\ -1'5x+y=0 \\ 0'5x-y+0'5z=0 \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 27000 \\ -3 & 2 & 0 & 0 \\ 1 & -2 & 1 & 0 \end{array}\right)$$

Por Cramer

$$\begin{vmatrix} 1 & 1 & 1 \\ -3 & 2 & 0 \\ 1 & -2 & 1 \end{vmatrix} = \cancel{2}+6-\cancel{2}+3=\boxed{9}$$

$$x=\frac{\begin{vmatrix} 27000 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & -2 & 1 \end{vmatrix}}{9}=\frac{54.000}{9}=\boxed{6.000}$$

$$y=\frac{\begin{vmatrix} 1 & 27.000 & 1 \\ -3 & 0 & 0 \\ 1 & 0 & 1 \end{vmatrix}}{9}=\frac{81000}{9}=$$

$\boxed{y=9.000}$

$$z=\frac{\begin{vmatrix} 1 & 1 & 27.000 \\ -3 & 2 & 0 \\ 1 & -2 & 0 \end{vmatrix}}{9}=\frac{108.000}{9}$$

$\boxed{z=12.000}$

7.-
$$\left.\begin{array}{l} 2y-x+z=3 \\ 3y-2x=5 \\ y-2\cancel{x}+2z=1-\cancel{x} \end{array}\right\} \left.\begin{array}{r} -x+2y+z=3 \\ -2x+3y=5 \\ y+2z=1 \end{array}\right\} A=\begin{pmatrix} -1 & 2 & 1 \\ -2 & 3 & 0 \\ 0 & 1 & 2 \end{pmatrix} \longrightarrow |A|=-6-2+8=0$$

$rg(A)\leq 2$

Cogemos un menor 2×2

$$\begin{vmatrix} -1 & 2 \\ -2 & 3 \end{vmatrix}=-3+4=1\neq 0 \Rightarrow rg(A)=2$$

$$|A^*|=\begin{vmatrix} -1 & 2 & 3 \\ -2 & 3 & 5 \\ 0 & 1 & 1 \end{vmatrix}=-3-6+4+5=0 \left.\begin{array}{l} \\ rg(A^*)\leq 2 \end{array}\right\} \Rightarrow rg(A^*)=2$$

Sistema Compatible Indeterminado

$$A^* = \left(\begin{array}{ccc|c} -1 & 2 & 1 & 3 \\ -2 & 3 & 0 & 5 \\ 0 & 1 & 2 & 1 \end{array}\right) \longrightarrow \left(\begin{array}{cc|c} -1 & 2 & 3-\lambda \\ -2 & 3 & 5 \end{array}\right)$$

$\boxed{z = \lambda}$

$$\begin{vmatrix} -1 & 2 \\ -2 & 3 \end{vmatrix} = -3+4 = 1$$

$$x = \begin{vmatrix} 3-\lambda & 2 \\ 5 & 3 \end{vmatrix} = 9-3\lambda-10 = \boxed{-1-3\lambda}$$

$$y = \begin{vmatrix} -1 & 3-\lambda \\ -2 & 5 \end{vmatrix} = -5+6-2\lambda = \boxed{1-2\lambda}$$

8.- x = cantidad invertida en A
y = " " " B
z = " " " C

$$\left.\begin{array}{l} x+y+z = 15.000 \\ y = \frac{1}{2}(x+z) \\ x = 0'25z \end{array}\right\} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 15.000 \\ 0'5 & -1 & 0'5 & 0 \\ 1 & 0 & -0'25 & 0 \end{array}\right)$$

GAUSS

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 15.000 \\ 1 & -2 & 1 & 0 \\ 4 & 0 & -1 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 15.000 \\ 0 & -3 & 0 & -15.000 \\ 0 & -4 & -5 & -60.000 \end{array}\right) \sim \begin{array}{c} \begin{array}{ccc} x & z & y \end{array} \\ \left(\begin{array}{ccc|c} 1 & 1 & 1 & 15.000 \\ 0 & 0 & -3 & -15.000 \\ 0 & -5 & -4 & -60.000 \end{array}\right) \end{array}$$

$$\sim \begin{array}{c} \begin{array}{ccc} x & z & y \end{array} \\ \left(\begin{array}{ccc|c} 1 & 1 & 1 & 15.000 \\ 0 & -5 & -4 & -60.000 \\ 0 & 0 & -3 & -15.000 \end{array}\right) \end{array} \longrightarrow -3y = -15.000$$

$\boxed{y = 5000}$

$-5z - 4y = -60.000$
$5z = 60000 - 4y$
$5z = 40.000$
$\boxed{z = 8000}$

$x = 15.000 - 5000 - 8000$
$\boxed{x = 2.000}$

9.- $\left.\begin{array}{l} 4x - 3y + 3 = 6 \\ x + 5y = 2 \end{array}\right\} \left.\begin{array}{l} 4x - 3y = 3 \\ x + 5y = 2 \end{array}\right\} \left(\begin{array}{cc|c} 4 & -3 & 3 \\ 1 & 5 & 2 \end{array}\right)$

$$\begin{vmatrix} 4 & -3 \\ 1 & 5 \end{vmatrix} = 20+3 = 23$$

$$x = \frac{\begin{vmatrix} 3 & -3 \\ 2 & 5 \end{vmatrix}}{23} = \frac{15+6}{23} = \boxed{\frac{21}{23}}$$

$$y = \frac{\begin{vmatrix} 4 & 3 \\ 1 & 2 \end{vmatrix}}{23} = \frac{8-3}{23} = \boxed{\frac{5}{23}}$$

10.- $AX + C = B$

$AX = B - C$

$A^{-1} \cdot AX = A^{-1}(B - C)$

$X = A^{-1} \cdot (B - C)$

$|A| = -6$

$Adj(A) = \begin{pmatrix} 0 & -2 \\ -3 & -1 \end{pmatrix}$

$A^{-1} = -\frac{1}{6} \cdot \begin{pmatrix} 0 & -3 \\ -2 & -1 \end{pmatrix}$

$$X = -\frac{1}{6}\begin{pmatrix} 0 & -3 \\ -2 & -1 \end{pmatrix} \cdot \begin{pmatrix} 2 & 2 \\ -3 & 3 \end{pmatrix} = -\frac{1}{6}\begin{pmatrix} 9 & -9 \\ -1 & -7 \end{pmatrix} = \boxed{\begin{pmatrix} -9/6 & 9/6 \\ 1/6 & 7/6 \end{pmatrix}}$$

11.- $\left.\begin{array}{l} x + \cancel{y} - \cancel{y} + 2z = 0 \\ 2x - 4y + y = 4 \\ x + y + y - z = 3 \end{array}\right\} \quad \left.\begin{array}{l} x + 2z = 0 \\ 2x - 3y = 4 \\ x + 2y - z = 3 \end{array}\right\} \quad \left(\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 2 & -3 & 0 & 4 \\ 1 & 2 & -1 & 3 \end{array}\right)$

Por Cramer

$$|A| = \begin{vmatrix} 1 & 0 & 2 \\ 2 & -3 & 0 \\ 1 & 2 & -1 \end{vmatrix} = 3 + 8 + 6 = 17$$

$$x = \frac{\begin{vmatrix} 0 & 0 & 2 \\ 4 & -3 & 0 \\ 3 & 2 & -1 \end{vmatrix}}{17} = \frac{16 + 18}{17} = \frac{34}{17} = 2$$

$$y = \frac{\begin{vmatrix} 1 & 0 & 2 \\ 2 & 4 & 0 \\ 1 & 3 & -1 \end{vmatrix}}{17} = \frac{-4 + 12 - 8}{17} = \boxed{0}$$

$$z = \frac{\begin{vmatrix} 1 & 0 & 0 \\ 2 & -3 & 4 \\ 1 & 2 & 3 \end{vmatrix}}{17} = \frac{-9 - 8}{17} = \boxed{-1}$$

12.- x = nº electricistas

y = nº mecánicos

$f(x, y) = 250x + 200y$

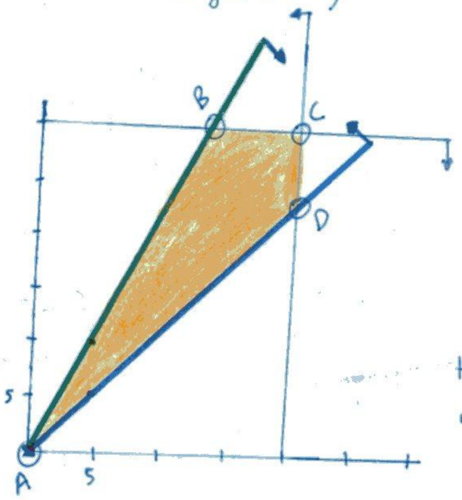
$\left.\begin{array}{l} y \geq x \\ y \leq 2x \\ 0 \leq x \leq 20 \\ 0 \leq y \leq 30 \end{array}\right\}$

$y = x$

x	y
0	0
5	5

$y = 2x$

x	y
0	0
5	10



$A(0,0) \rightarrow f(0,0) = 0$

$B(15, 30) \rightarrow f(15,30) = 9750$

$\boxed{C(20,30) \rightarrow f(20,30) = 11.000}$

$D(20,20) \rightarrow f(20,20) = 9.000$

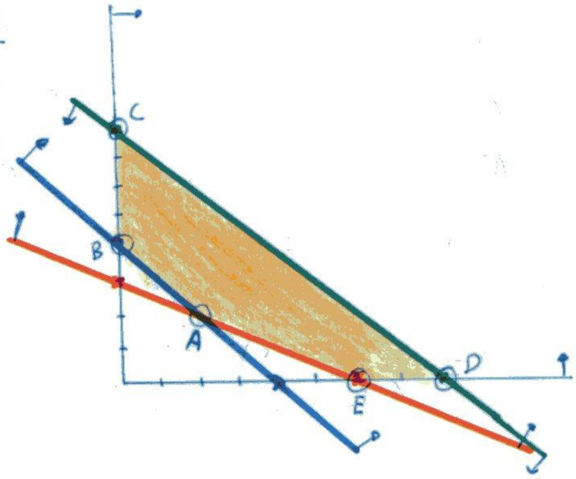
Habrán 20 electricistas y 30 mecánicos, con un beneficio de 11.000 €

13.- $\left.\begin{array}{l} x+y\le 8 \\ x+y\ge 4 \\ x+2y\ge 6 \\ x\ge 0 \\ y\ge 0 \end{array}\right\}$ $\begin{array}{l} \to y=8-x \\ \to y=4-x \\ \to y=\dfrac{6-x}{2} \end{array}$

x	y
0	8
8	0

x	y
0	4
4	0

x	y
0	3
6	0



a)

A

$\left.\begin{array}{l} y=4-x \\ y=\dfrac{6-x}{2} \end{array}\right\}$ $4-x=\dfrac{6-x}{2}$

$8-2x=6-x$

$-x=-2$

$x=2\to y=2$

$\boxed{A\,(2,2)}$

B $\boxed{B\,(0,4)}$

C $\boxed{C\,(0,8)}$

D $\boxed{D\,(8,0)}$

E $\boxed{E\,(6,0)}$

b) $f(x,y)=3x+2y$

$f(2,2)=10$

$\boxed{f(0,4)=8}\to$ Valor mínimo

$f(0,8)=16$

$f(8,0)=24$

$f(6,0)=18$

14.- x = nº motos nuevas

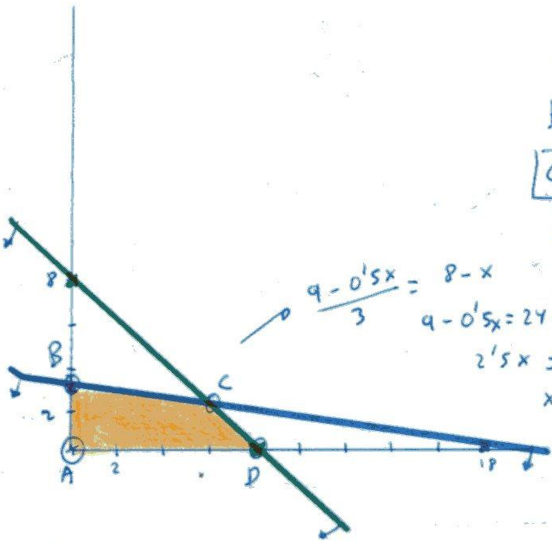
y = nº motos usadas

$f(x,y)=15x+27y$

$\left.\begin{array}{l} 0'5x+3y\le 9 \\ x+y\le 8 \\ x\ge 0 \\ y\ge 0 \end{array}\right\}$ $\begin{array}{l} \to y=\dfrac{9-0'5x}{3} \\ \to y=8-x \end{array}$

x	y
0	3
18	0

x	y
0	8
8	0



$\dfrac{9-0'5x}{3}=8-x$

$9-0'5x=24-3x$

$2'5x=15$

$x=6$

$A\,(0,0)\to f(0,0)=0$

$B\,(0,3)\to f(0,3)=81$

$\boxed{C\,(6,2)\to f(6,2)=144}$

$D\,(8,0)\to f(8,0)=120$

Debe revisar 6 motos nuevas y 2 usadas para obtener unos beneficios de 144 €

15.-

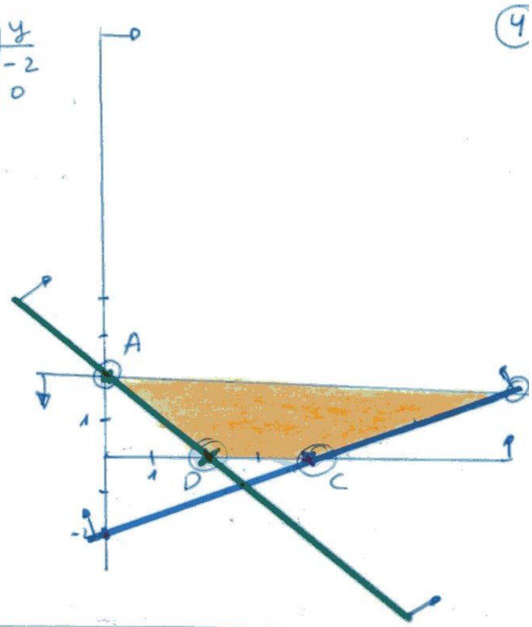
$$\left.\begin{array}{l} x-2y\le 4 \\ x+y\ge 2 \\ x\ge 0 \\ 0\le y\le 2 \end{array}\right\}$$

$y=\dfrac{-4+x}{2} \longrightarrow$

x	y
0	-2
4	0

$y=2-x \longrightarrow$

x	y
0	2
2	0



$A(0,2)\rightarrow f(0,2)=4$

$\boxed{B(8,2)\rightarrow f(8,2)=12}$

$C(4,0)\rightarrow f(4,0)=4$

$D(2,0)\rightarrow f(2,0)=2$

16.- Si $0\le x<3 \rightarrow$ la función es continua por ser polinómica

- Si $3<x<5$
- Si $5<x\le 10$

Si $\underline{x=3}$

$f(3)=3^2-3\cdot 3=9-9=\boxed{0}$

$\lim\limits_{x\to 3^-} f(x)=2\cdot 3-1=\boxed{5}$

$\lim\limits_{x\to 3^+} f(x)=\boxed{0}$

La función es discontinua en x = 3. Discontinuidad de salto finito

Si $\underline{x=5}$ $f(5)=10$

$\lim\limits_{x\to 5^-} f(x)=10$

$\lim\limits_{x\to 5^+} f(x)=m-20$

$10=m-20$

$\boxed{m=30} \rightarrow$ Si $m=30$, la función es continua en $\underline{x=5}$

17.- $P(B/A)=\dfrac{P(A\cap B)}{P(A)}=\dfrac{P(A\cap B)}{\frac{7}{10}}=\dfrac{2}{3}$

$P(A\cap B)=\dfrac{14}{30}$

$P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\dfrac{4}{10}=\dfrac{7}{10}+P(B)-\dfrac{14}{30}$

$P(B)=\dfrac{-3}{10}+\dfrac{14}{30}$

$P(B)=\dfrac{-9+14}{30}=\dfrac{5}{30}=\boxed{\dfrac{1}{6}}$

18.- - Si $x < 1 \longrightarrow$ La función es continua por ser polinómica

- Si $1 < x < 3 \longrightarrow x - 1 = 0$

$x = 1 \rightarrow$ como no pertenece al intervalo, la función es continua en dicho intervalo

- Si $x > 3 \longrightarrow$ la función es continua al ser polinómica

- $\boxed{x = 1}$

$$\left.\begin{array}{l} f(1) = 2 \\ \lim_{x \to 1^-} f(x) = 2 \\ \lim_{x \to 1^+} f(x) = \frac{4}{0} = \infty \end{array}\right\} \text{Discontinuidad inevitable de salto infinito en } x = 1$$

- $\boxed{x = 3}$

$$\left.\begin{array}{l} f(3) = \nexists \\ \lim_{x \to 3^-} f(x) = \frac{12}{2} = 6 \\ \lim_{x \to 3^+} f(x) = 6 - 3 = 3 \end{array}\right\} \text{Discontinuidad de salto finito en } \boxed{x = 3}$$

19.- a) $y = x^3 - 2x^2 + x$

- Dominio: $\mathbb{R}$
- Ptos de corte

$OX \rightarrow y = 0 \rightarrow x^3 - 2x^2 + x = 0$

$x(x^2 - 2x + 1) = 0$

$x = 0$; $x^2 - 2x + 1 = 0$, $x = 1$

$\boxed{(0,0)}\boxed{(1,0)}$

$OY \rightarrow x = 0 \rightarrow y = 0 \Rightarrow \boxed{(0,0)}$

- No hay asíntotas, ya que es polinómica
- Rama parabólica

$\lim_{x \to +\infty} f(x) = +\infty$ ↗

$\lim_{x \to -\infty} f(x) = -\infty$ ↙

- Monotonía

$f'(x) = 3x^2 - 4x + 1 = 0$

$\boxed{x = 1}$, $\boxed{x = \frac{1}{3}}$

	$-\infty$		$1/3$		1		$+\infty$
$f'(x)$		+		−		+	
		↗		↘		↗	

Creciente: $\left(-\infty, \frac{1}{3}\right) \cup (1, +\infty)$

Decreciente: $\left(\frac{1}{3}, 1\right)$

- Extremos locales

$f''(x) = 6x - 4$

$f''\left(\frac{1}{3}\right) = -2 < 0 \rightarrow$ Máximo $\boxed{\left(\frac{1}{3}, \frac{4}{27}\right)}$

$f''(1) = 2 > 0 \rightarrow$ Mínimo $\boxed{(1,0)}$

- Curvatura

$f''(x) = 6x - 4 = 0$

$x = \frac{4}{6} = \frac{2}{3}$

$\xleftarrow{\quad - \quad} \; \frac{2}{3} \; \xrightarrow{\quad + \quad} f''(x)$

$-\infty \qquad \frac{2}{3} \qquad +\infty$

$\cap \qquad \cup$

Cóncava hacia abajo $\left(-\infty, \frac{2}{3}\right)$

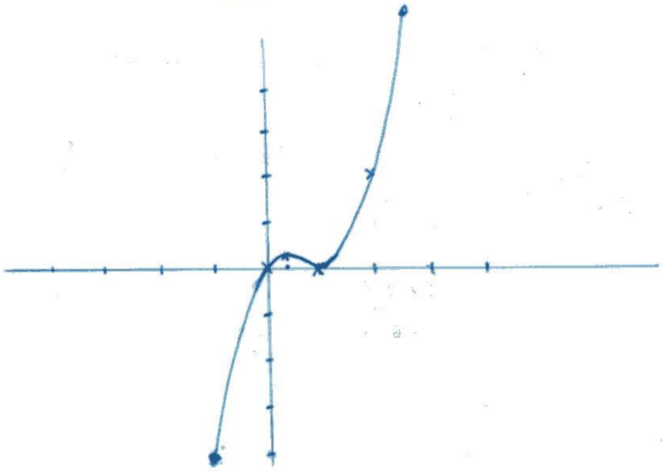
Cóncava hacia arriba $\left(\frac{2}{3}, +\infty\right)$

- Ptos de inflexión:

$f'''(x) = 6 \neq 0 \longrightarrow$ pto $\boxed{\left(\frac{2}{3}, \frac{2}{27}\right)}$

Tabla de valores

x	y
2	2
-1	-4



b) $f(x) = \frac{x}{(x-3)^2} = \frac{x}{x^2 - 6x + 9}$

- Dominio: $\mathbb{R} - \{3\}$
- Ptos de corte

$OX \rightarrow y = 0 \rightarrow x = 0 \rightarrow \boxed{(0,0)}$

$OY \rightarrow x = 0 \rightarrow y = 0 \rightarrow \boxed{(0,0)}$

- Asíntotas
- Horizontales

$y = \lim_{x \to +\infty} f(x) = \frac{\infty}{\infty} = 0$

$y = \lim_{x \to -\infty} f(x) = \frac{\infty}{\infty} = 0$

Comportamiento $\boxed{y = 0}$

$\frac{x}{x^2 - 6x + 9} - 0 =$ $+\infty$ (+) Por enci; $-\infty$ (−) Por debajo

- Verticales

$\lim_{x \to 3} f(x) = \frac{3}{0} = \begin{cases} \lim_{x \to 3^-} f(x) = \frac{+}{+} = +\infty & \uparrow \text{ Hacia arriba} \\ \lim_{x \to 3^+} f(x) = \frac{+}{+} = +\infty & \uparrow \text{ Hacia arriba} \end{cases}$

$\boxed{x = 3}$

- Monotonía

$$f'(x) = \frac{(x-3)^{\cancel{2}\,1} - x \cdot 2\,(\cancel{x-3})}{(x-3)^{\cancel{4}\,3}} = \frac{x-3-2x}{(x-3)^3} = \frac{-x-3}{(x-3)^3}$$

$-x-3=0$

$\boxed{x=-3}$

$f'(x)$: $(-\infty, -3)$: $-$; $(-3, 3)$: $+$; $(3, +\infty)$: $-$

Creciente: $(-3, 3)$

Decreciente: $(-\infty, -3) \cup (3, +\infty)$

- Máximos y mínimos:

Mínimo: $\boxed{\left(-3, -\frac{1}{12}\right)}$

En $x=3$ no hay máximo, puesto que no pertenece al dominio

- Curvatura

$$f''(x) = \frac{-(\cancel{x-3})^{\cancel{3}\,1} - (-x-3) \cdot 3\,(\cancel{x-3})^{\cancel{2}}}{(x-3)^{\cancel{6}\,4}} = \frac{-(x-3)+3x+9}{(x-3)^4} = \frac{2x+12}{(x-3)^4}$$

$2x+12=0$

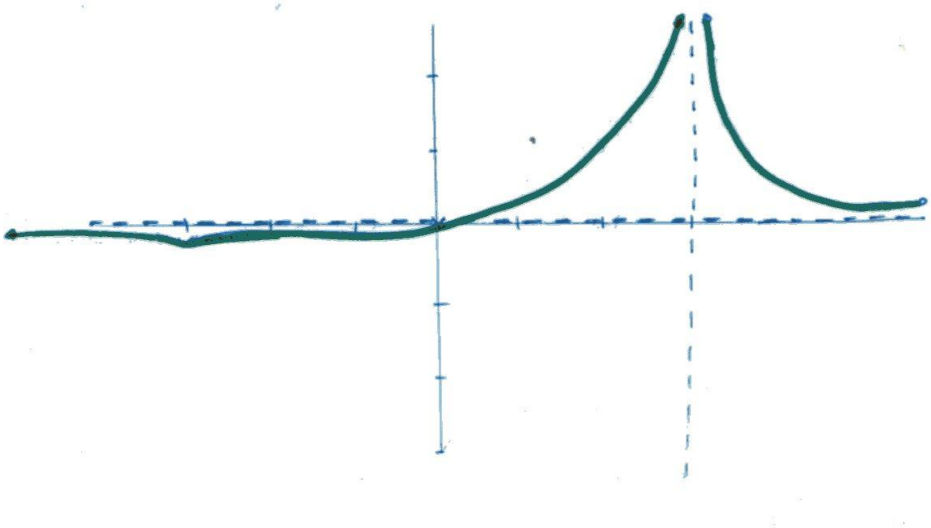
$\boxed{x=-6}$

$f''(x)$: $(-\infty, -6)$: $-$; $(-6, 3)$: $+$; $(3, +\infty)$: $+$

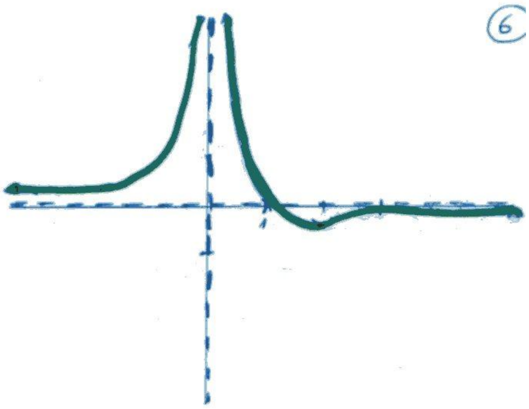
Cóncava hacia arriba: $(-6, 3) \cup (3, +\infty)$

Cóncava hacia abajo: $(-\infty, -6)$

- Ptos de inflexión: $\boxed{\left(-6, -\frac{2}{27}\right)}$



c) $f(x) = \frac{1-x}{x^2}$



- Dominio: $\mathbb{R} - \{0\}$
- Ptos de corte
  $OX \rightarrow y = 0 \rightarrow x = 1 \Rightarrow \boxed{(1, 0)}$
  $OY \rightarrow x = 0 \rightarrow y = \nexists$
- Asíntotas
  - Horizontales: $y = \lim_{x \to +\infty} f(x) = \frac{\infty}{\infty} = 0^- \longrightarrow$ Por debajo
    $\boxed{y = 0}$
    $y = \lim_{x \to -\infty} f(x) = \frac{\infty}{\infty} = 0^+ \longrightarrow$ Por encima
  - Verticales
    $\lim_{x \to 0} f(x) = \frac{1}{0} = \begin{cases} \lim_{x \to 0^-} f(x) = \frac{+}{+} = +\infty \uparrow \text{ Para arriba} \\ \lim_{x \to 0^+} f(x) = \frac{+}{+} = +\infty \uparrow \text{ Para arriba} \end{cases}$
    $\boxed{x = 0}$
- Crecimiento y decrecimiento:

$$f'(x) = \frac{-(x^2) - (1-x) \cdot 2x}{x^4} = \frac{-x^2 - 2x + 2x^2}{x^4} = \frac{x^2 - 2x}{x^4} = \frac{x-2}{x^3} = 0$$

$$x - 2 = 0 \longrightarrow \boxed{x = 2}$$

$-\infty$		0		2		$+\infty$	$f'(x)$
	$+$		$-$		$+$		
	$\nearrow$		$\searrow$		$\nearrow$		

Creciente: $(-\infty, 0) \cup (2, +\infty)$
Decreciente: $(0, 2)$

- Máximos y mínimos: - Máximo: no existe, puesto que $x = 0$ no pertenece al dominio
  - Mínimo: $\boxed{\left(2, -\frac{1}{4}\right)}$
- Curvatura

$$f''(x) = \frac{x^3 - (x-2) \cdot 3x^2}{x^6} = \frac{x^3 - 3x^3 + 6x^2}{x^6} = \frac{-2x^3 + 6x^2}{x^6} = \frac{-2x + 6}{x^4} = 0$$

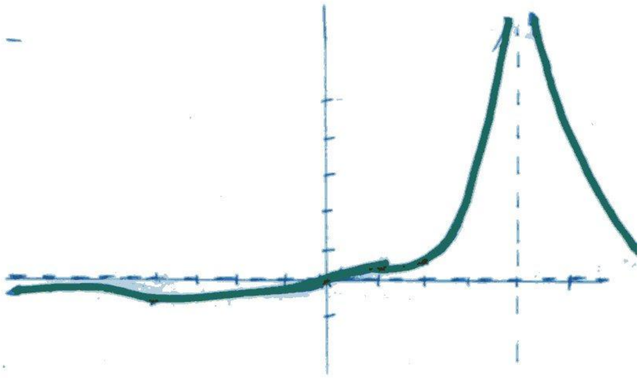
$-2x + 6 = 0$
$\boxed{x = 3}$

$-\infty$		0		3		$+\infty$	$f''(x)$
	$+$		$+$		$-$		
	$\cup$		$\cup$		$\cap$		

Cóncava: $(-\infty, 0) \cup (0, 3)$
Convexa: $(3, +\infty)$

- Ptos inflexión
  $\boxed{\left(3, -\frac{2}{9}\right)}$

d) $f(x) = \dfrac{x}{(x-4)^2} = \dfrac{x}{x^2-8x+16}$



- Dominio: $\mathbb{R} - \{4\}$
- Ptos corte

$OX \to y=0 \to x=0 \Rightarrow \boxed{(0,0)}$

$OY \to x=0 \to y=0 \Rightarrow \boxed{(0,0)}$

x	y
5	5
2	0'5

- Asíntotas
- Horizontales

$y = \lim_{x \to +\infty} f(x) = \frac{\infty}{\infty} = \boxed{0^+}$ Por encima

$y = \lim_{x \to -\infty} f(x) = \frac{\infty}{\infty} = \boxed{0^-}$ Por debajo

$\boxed{y=0}$

- Verticales

$\lim_{x \to 4} f(x) = \frac{4}{0} =$ $\begin{cases} \lim_{x \to 4^-} f(x) = \frac{+}{+} = +\infty \ \uparrow \text{ Para arriba} \\ \lim_{x \to 4^+} f(x) = \frac{+}{+} = +\infty \ \uparrow \text{ Para arriba} \end{cases}$

$\boxed{x=4}$

- Monotonía

$f'(x) = \dfrac{(x-4)^{\cancel{2}\,1} - x \cdot 2\cancel{(x-4)}}{(x-4)^{\cancel{4}\,3}} = \dfrac{x-4-2x}{(x-4)^3} = \dfrac{-x-4}{(x-4)^3} = 0 \longrightarrow -x-4=0$

$\boxed{x=-4}$

$-\infty$ (−) -4 (+) 4 (−) $+\infty$

Creciente: $(-4, 4)$

Decreciente: $(-\infty, -4) \cup (4, +\infty)$

- Máximos y mínimos: Mínimo: $\boxed{\left(-4, -\frac{1}{16}\right)}$

Máximo: No hay

- Curvatura:

$f''(x) = \dfrac{-(x-4)^{\cancel{3}\,1} - (-x-4)\,3\,\cancel{(x-4)^2}}{(x-4)^{\cancel{6}\,4}} = \dfrac{-(x-4)+3x+12}{(x-4)^4} = \dfrac{2x+16}{(x-4)^4} = 0$

$2x+16=0$

$\boxed{x=-8}$

$-\infty$ (−) -8 (+) 4 (+) $+\infty$

Cóncava: $(-8, 4) \cup (4, +\infty)$

Convexa: $(-\infty, -8)$

Ptos de inflexión

$\boxed{\left(-8, \frac{-1}{18}\right)}$

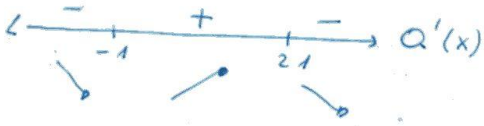
20.- $Q(x) = (x+1)^2(32-x) \longrightarrow$ Maximizar

a) $Q(x) = (x^2 + 1 + 2x)(32 - x) = 32x^2 + 32 + 64x - x^3 - x - 2x^2 =$

$= -x^3 + 30x^2 + 63x + 32$

$Q'(x) = -3x^2 + 60x + 63 = 0$

$\boxed{x = 21}$ $\boxed{x = -1}$



$Q''(x) = -6x + 60$

$Q''(21) = -66 < 0 \rightarrow$ Máxima producción con $\boxed{21^{\circ}}$ de temperatura

La producción será de $\boxed{Q(21) = 5324 \text{ Kg}}$

21.- a) $\int_{-2}^{2} 2x^2 - x - 3\, dx = \left[\frac{2}{3}x^3 - \frac{x^2}{2} - 3x\right]_{-2}^{2} = \left|\left(\frac{16}{3} - 2 - 6\right) - \left(-\frac{16}{3} - 2 + 6\right)\right| =$

$= \left|-\frac{8}{3} + \frac{4}{3}\right| = \boxed{\frac{4}{3}}$

b) Ptos de corte $\quad 2x^2 - x - 3 = 0 \longrightarrow \boxed{x = \frac{3}{2}}$ $\boxed{x = -1}$

$\int_{-2}^{-1} 2x^2 - x - 3\, dx + \int_{-1}^{3/2} 2x^2 - x - 3\, dx + \int_{3/2}^{2} 2x^2 - x - 3\, dx =$

$\left|\left[\frac{2}{3}x^3 - \frac{x^2}{2} - 3x\right]_{-2}^{-1}\right| + \left|\left[\frac{2}{3}x^3 - \frac{x^2}{2} - 3x\right]_{-1}^{3/2}\right| + \left|\left[\frac{2}{3}x^3 - \frac{x^2}{2} - 3x\right]_{3/2}^{2}\right| =$

$= \left|\frac{11}{6} - \left(-\frac{4}{3}\right)\right| + \left|-\frac{27}{8} - \frac{11}{6}\right| + \left|-\frac{8}{3} - \left(-\frac{27}{8}\right)\right| = \left|\frac{19}{6}\right| + \left|-\frac{125}{24}\right| + \left|\frac{17}{24}\right| =$

$= \frac{19}{6} + \frac{125}{24} + \frac{17}{24} = \boxed{\frac{109}{12}}$

22.- $f(x) = -x^2+40x+84$

a) $f(x)=0 \longrightarrow -x^2+40x+84=0$

$x = \begin{cases} \boxed{42} \\ \boxed{-2} \text{ NO VALE} \end{cases}$ Deben transcurrir 42 días

b) $TV[0,5] = f(5)-f(0) = 259-84 = \boxed{175}$

c) $f'(x) = -2x+40 = 0$

$-2x=-40$

$x=20$

$f'(x)$: 0 $+$ 20 $-$ $+\infty$; $\nearrow$ $\searrow$

Deja de crecer a partir del día $\boxed{20}$

23.- $\boxed{x=2}$ $f(2)=7$

a)

$\left.\begin{array}{l} \lim\limits_{x\to 2^-} f(x) = 7 \\ \lim\limits_{x\to 2^+} f(x) = 1-2a \end{array}\right\}$ $1-2a=7$

$-2a=6$

$\boxed{a=-3}$

b) $\int_{-1}^{3} f(x) = \int_{-1}^{2}(2x^2-1)\,dx + \int_{2}^{3}(1+3x)\,dx = \left|\left[\frac{2x^3}{3}-x\right]_{-1}^{2}\right| + \left|\left[x+\frac{3x^2}{2}\right]_{2}^{3}\right| =$

$= \left|\frac{10}{3}-\frac{1}{3}\right| + \left|\frac{33}{2}-\cancel{\frac{10}{3}}\,8\right| = \frac{9}{3} + \cancel{\frac{79}{6}}\,17/2 = \boxed{\frac{23}{2}\,u^2}$

24.- a) $f(x) = x^2+3x+5 \longrightarrow f'(x) = 2x+3$

$f'(-1) = -2+3 = \boxed{1=m}$ $f(-1) = 1-3+5=3$

$\left.\right\}$ $y-3 = 1(x+1)$

$y-3=x+1$

$\boxed{y=x+4}$

b) $\int_0^3 (x^2+3x+5)\,dx = \left|\left[\frac{x^3}{3}+\frac{3x^2}{2}+5x\right]_0^3\right| = \left|9+\frac{27}{2}+15\right| = \boxed{\frac{75}{2}}\,u^2$

$x^2+3x+5=0$

$x =$ No hay ptos de corte

25.- G = "usar gafas" $\quad P(G) = \frac{1}{5} = 0'2$

L = "usar lentillas" $\quad P(\bar{G} \cap \bar{L}) = 0'32$

$P(G \cap \bar{L}) = 0'12$

¿$P(L)$?

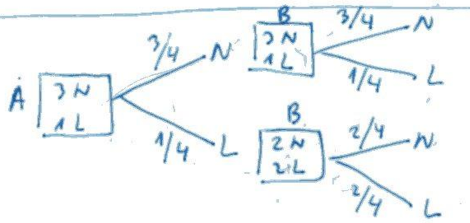
$$P(\bar{G} \cap \bar{L}) = P(\overline{G \cup L}) = 1 - P(G \cup L) = 1 - [P(G) + P(L) - P(G \cap L)] =$$

$$P(G \cap \bar{L}) = P(G) - P(G \cap L) = \frac{1}{5} - P(G \cap L) = 0'12$$

$$\underline{P(G \cap L) = 0'08}$$

$$1 - [0'2 + P(L) - 0'08] =$$
$$= 1 - 0'12 - P(L) = 0'32$$
$$\boxed{P(L) = 0'56} = 56\%$$

26.-



a) $P(L) = \frac{3}{4} \cdot \frac{1}{4} + \frac{1}{4} \cdot \frac{2}{4} = \frac{3}{16} + \frac{2}{16} = \boxed{\frac{5}{16}}$

b) $P(L_A / L_B) = \frac{P(L_A \cap L_B)}{P(L_B)} = \frac{1/4 \cdot 2/4}{5/16} = \boxed{\frac{2}{5}}$

27.- $P(L) = 0'7 \qquad P(C/L) = \frac{P(C \cap L)}{P(L)} =$

$P(C) = 0'2$

$$P(L/\bar{C}) = 0'75 \rightarrow \frac{P(L \cap \bar{C})}{P(\bar{C})} = \frac{P(L) - P(L \cap C)}{P(\bar{C})} = \frac{0'7 - P(L \cap C)}{0'8} = 0'75$$

$$\boxed{P(L \cap C) = 0'1}$$

a) $P(C/L) = \frac{P(C \cap L)}{P(L)} = \frac{0'1}{0'7} = \boxed{0'1428}$

b) $P(\bar{L} \cap \bar{C}) = P(\overline{L \cup C}) = 1 - P(L \cup C) = 1 - [P(L) + P(C) - P(L \cap C)] =$

$$= 1 - [0'7 + 0'2 - 0'1] = \boxed{0'2}$$

28.- $P(A \cap B) = \frac{1}{5}$

$P(B) = \frac{1}{5}$

$P(A/B) = \frac{2}{3} \longrightarrow P(A/B) = P(A) = \boxed{\frac{2}{3}}$ Ya que $\boxed{A \subset B}$

incluido

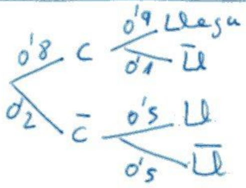
29.- a) $\boxed{P(B) = \frac{55}{95}}$

	N	D	
A	30	10	40
B	40	15	55
TOT	70	25	95

b) $P(D \cap A) = \boxed{\frac{10}{95}}$

c) $P(N/A) = \boxed{\frac{30}{40}}$

30.-

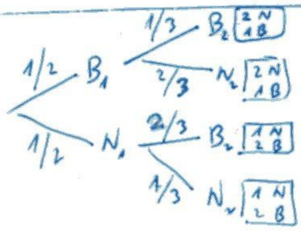


a) $P(\text{Llegar}) = 0'8 \cdot 0'9 + 0'2 \cdot 0'5 = \boxed{0'82}$

b) $P(C/Ll) = \frac{P(C \cap Ll)}{P(Ll)} = \frac{0'8 \cdot 0'9}{0'82} = \boxed{0'878}$

c) $P(\bar{C}/\overline{Ll}) = \frac{P(\bar{C} \cap \overline{Ll})}{P(\overline{Ll})} = \frac{0'2 \cdot 0'5}{0'18} = \boxed{0'\hat{5}}$

31.-



a) $P(B_2/N_1) = \boxed{\frac{2}{3}}$

b) $P(B_2) = \frac{1}{2} \cdot \frac{1}{3} + \frac{1}{2} \cdot \frac{2}{3} = \frac{3}{6} = \boxed{\frac{1}{2}}$

32.- x = manzanas $\rightarrow P(M) = \frac{1}{3}$ $\qquad P(N_1 \cap N_2) = 0'2$

$2x$ = naranjas $\rightarrow P(N) = \frac{2}{3}$ $\qquad P(M_1 \cap M_2) = 0'6$

$P(\text{al menos una } M) = 1 - P(\text{ninguna } M) = 1 - P(\bar{M}_1 \cap \bar{M}_2) = 1 - P(\overline{M_1 \cup M_2}) =$

$= \cancel{1} - [\cancel{1} - P(M_1 \cup M_2)] = P(M_1 \cup M_2) = P(M_1) + P(M_2) - P(M_1 \cap M_2) =$

$= \frac{1}{3} + \frac{1}{3} - 0'6 = \boxed{0'0\hat{6}}$

33.- $P(\bar{M} \cap \bar{R}) = 0'2$

$P(\bar{M}) = 0'4$

$P(R/M) = \frac{1}{6}$

a) $P(M/R)$

$P(\bar{M} \cap \bar{R}) = P(\overline{M \cup R}) = 1 - P(M \cup R) = 1 - [P(M) + P(R) - P(M \cap R)$

$P(R/M) = \frac{P(M \cap R)}{P(M)} = \frac{P(M \cap R)}{0'6} = \frac{1}{6}$

$\boxed{P(M \cap R) = 0'1}$

$0'2 = 1 - [0'6 + P(R) - 0'1]$

$\boxed{P(R) = 0'3}$

$P(M/R) = \frac{P(M \cap R)}{P(R)} = \frac{0'1}{0'3} = \boxed{\frac{1}{3}}$

b) $P(R) = 0'3 \rightarrow 30\%$

30% de 900 = $\boxed{270}$

34.- $P(A\cup B)=0'8$, $P(B/A)=0'5$, $P(B)=0'4$

$$P(A\cup B)=P(A)+P(B)-P(A\cap B)$$

$$0'8=P(A)+0'4-P(A\cap B)$$

$$P(B/A)=\frac{P(A\cap B)}{P(A)}$$

$$0'5=\frac{P(A\cap B)}{P(A)}$$

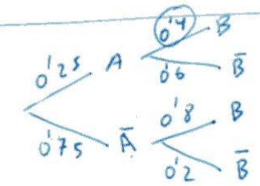
$$0'5\cdot P(A)=P(A\cap B)$$

$$0'4=P(A)-0'5\cdot P(A)$$

$$0'4=0'5\cdot P(A)$$

$$\boxed{P(A)=0'8}\longrightarrow P(A\cap B)=0'8\cdot 0'5=\boxed{0'4}$$

35.-



$P(B)=0'7$

$P(A)=0'25$

$$P(B/A)=\frac{P(A\cap B)}{P(A)}=\frac{0'1}{0'25}=0'4$$

$$P(\bar{A}\cap\bar{B})=0'75\cdot 0'2=0'15=1-P(A\cup B)$$

$$P(A\cup B)=0'85=P(A)+P(B)-P(A\cap B)$$

$$0'85=0'25+0'7-P(A\cap B)$$

$$P(A\cap B)=0'1$$