Repaso Aplicaciones Derivadas

2.- - En $x=1$ tiene pto inflexión $\Rightarrow \boxed{f''(1)=0}$

- Tangente horizontal en $x=1 \Rightarrow m=0 \rightarrow$ Pendiente

$\boxed{f'(1)=0}$

$f'(x)=3x^2+2ax+b \longrightarrow f'(1)=3+2a+b=0$

$f''(x)=6x+2a \longrightarrow f''(1)=6+2a=0$

$\rightarrow 2a=-6$

$\boxed{a=-3}$

$3-6+b=0$

$\boxed{b=3}$

3.- $f'(x)=3x^2+2ax+b$ $\qquad f(x)=x^3+ax^2+bx+c$

$f''(x)=6x+2a$

En $(1,1)$ tiene pto inflexión $\longrightarrow f''(1)=0 \Rightarrow \boxed{6+2a=0}$

Pasa por $(1,1) \longrightarrow f(1)=1 \Longrightarrow \boxed{1+a+b+c=1}$

Tangente horizontal en $x=1 \longrightarrow f'(1)=0 \Rightarrow \boxed{3+2a+b=0}$

pendiente

$\boxed{a=-3}$

$2a+b=-3$

$b=-3+6$

$\boxed{b=3}$

$a+b+c=0$

$-3+3+c=0$

$\boxed{c=0}$

4.- $f(x)=x^3+ax^2+bx+c$ / $f'(x)=3x^2+2ax+b$ / $f''(x)=6x+2a$

- Pasa por $(0,3) \Rightarrow f(0)=3 \Rightarrow \boxed{c=3}$
- Extremo en $x=1 \Rightarrow f'(1)=0 \Rightarrow \boxed{3+2a+b=0}$
- Extremo en $x=3 \Rightarrow f'(3)=0 \Rightarrow \boxed{27+6a+b=0}$

$2a+\cancel{b}=-3$

$-6a-\cancel{b}=27$

$-4a=24$

$\boxed{a=-6}$

$2\cdot(-6)+b=0$

$\boxed{b=12}$

$f''(1)=6+2\cdot(-6)=-6<0 \Rightarrow$ **Máximo en $x=1$**

$f''(3)=18+2(-6)=6>0 \Rightarrow$ **Mínimo en $x=1$**

5.- $f(x) = x^3 + ax^2 + bx + c$ / $f'(x) = 3x^2 + 2ax + b$ / $f''(x) = 6x + 2a$

- Tiene tangente horizontal en $x = -4 \Rightarrow f'(-4) = 0$
pendiente

$\Downarrow$

$\boxed{48 - 8a + b = 0}$

- Pto de inflexión en $x = -2 \Rightarrow f''(-2) = 0$

$\boxed{-12 + 2a = 0}$

- Pasa por $(1,1) \rightarrow f(1) = 1 \Rightarrow \boxed{1 + a + b + c = 1}$

$$\left.\begin{array}{l} 2a = 12 \\ -8a + b = -48 \\ a + b + c = 0 \end{array}\right\} \rightarrow \boxed{a = 6} \quad \boxed{b = 0} \quad \boxed{c = -6}$$

6.- $f(x) = ax^3 + bx^2 + cx + d$ / $f'(x) = 3ax^2 + 2bx + c$ / $f''(x) = 6ax + 2b$

- Pasa por $(1,0) \rightarrow f(1) = 0 \Rightarrow \boxed{a + b + c + d = 0}$
- Recta tg. paralela a $y = 2x$ en $x = 0$

$\Downarrow \quad \downarrow m = 2$

$f'(0) = 2 \Rightarrow \boxed{c = 2}$

- Extremo en $x = 1 \rightarrow f'(1) = 0 \Rightarrow \boxed{3a + 2b + c = 0}$
- Extremo en $x = 2 \rightarrow f'(2) = 0 \Rightarrow \boxed{12a + 4b + c = 0}$

$$\left.\begin{array}{l} a + b + d = -2 \\ 3a + 2b = -2 \\ 12a + 4b = -2 \end{array}\right\} > \begin{array}{r} -6a - \cancel{4b} = 4 \\ 12a + \cancel{4b} = -2 \\ \hline 6a = 2 \end{array}$$

$\boxed{a = \frac{2}{6} = \frac{1}{3}}$

$1 + 2b = -2$

$2b = -3$

$\boxed{b = -3/2}$

$\frac{1}{3} - \frac{3}{2} + d = -2$

$d = -2 - \frac{1}{3} + \frac{3}{2}$

$\boxed{d = \frac{5}{6}}$

$x = 1$

$f''(1) = 2 - 3 = -1 < 0 \Rightarrow$ MÁXIMO

$x = 2$

$f''(2) = 4 - 3 = 1 > 0 \Rightarrow$ MÍNIMO

7.- $f(x) = x^3 + ax^2 + bx + c$ / $f'(x) = 3x^2 + 2ax + b$ / $f''(x) = 6x + 2a$

- Corta al eje de abscisas en $x = -1 \Rightarrow$ Pasa $(-1, 0)$
  $y = 0$
  $\Downarrow$
  $\boxed{-1 + a - b + c = 0}$
- Pto inflexión $(2, 1) \longrightarrow f''(2) = 0 \Rightarrow \boxed{12 + 2a = 0}$
  $\Downarrow$
  $\boxed{a = -6}$
  $\downarrow$
  $f(2) = 1 \Rightarrow \boxed{8 + 4a + 2b + c = 1}$

$$\left.\begin{array}{l} -b + c = 7 \\ 2b + c = 17 \end{array}\right\} \begin{array}{l} b - \cancel{c} = -7 \\ 2b + \cancel{c} = 17 \\ \hline 3b = 10 \end{array} \Rightarrow \boxed{b = \frac{10}{3}}$$

$-\frac{10}{3} + c = 7.$

$c = 7 + \frac{10}{3}$

$\boxed{c = \frac{31}{3}}$

8.- $f(x) = ax^2 + bx + c$ / $f'(x) = 2ax + b$ / $f''(x) = 2a$

- Tg a la recta $y = 2x - 3$ en $(2, 1)$
  $\Downarrow$
  $m = f'(2) = 2$
  $\Downarrow$
  $\boxed{4a + b = 2}$

  $\to f(2) = 1$
  $\Downarrow$
  $\boxed{4a + 2b + c = 1}$
- Pasa $(5, -2) \Rightarrow f(5) = \boxed{25a + 5b + c = -2}$

$$\left.\begin{array}{l} 4a + 2b + c = 1 \\ 4a + b = 2 \\ 25a + 5b + c = -2. \end{array}\right\} \xrightarrow{\text{Gauss}} \left(\begin{array}{ccc|c} 4 & 2 & 1 & 1 \\ 4 & 1 & 0 & 2 \\ 25 & 5 & 1 & -2 \end{array}\right) \sim \left(\begin{array}{ccc|c} 4 & 2 & 1 & 1 \\ 0 & -1 & -1 & 1 \\ 0 & -30 & -21 & -33 \end{array}\right) \sim$$

$F_2 = F_2 - F_1$ $\quad$ $F_3 = F_3 - 30F_2$

$F_3 = 4F_3 - 25F_2$

$$\sim \left(\begin{array}{ccc|c} 4 & 2 & 1 & 1 \\ 0 & -1 & -1 & 1 \\ 0 & 0 & 9 & -63 \end{array}\right) \to 9c = -63$$

$\boxed{c = -7}$

$-b + 7 = 1$

$-b = -6$

$\boxed{b = 6}$

$4a + 12 - 7 = 1$

$4a = -4$

$\boxed{a = -1}$

Se podría hacer por Cramer también

9.- $f(x) = x^3 + ax^2 + bx + 2$ / $f'(x) = 3x^2 + 2ax + b$ / $f''(x) = 6x + 2a$

- Tg $y = 2x - 7$ a $x = 1 \Rightarrow f'(1) = 2$

  ↓ pendiente

$$\boxed{3 + 2a + b = 2}$$

- Al ser pto tangente $x = 1$, indica que pasa por la recta tg $\Rightarrow$ $y = 2 - 7$, $y = -5$ > $(1, -5)$

$$\Downarrow$$

$$f(1) = \boxed{1 + a + b + 2 = -5}$$

$$- \left.\begin{array}{l} 2a + \cancel{b} = -1 \\ a + \cancel{b} = -8 \end{array}\right\}$$

$\boxed{a = 7}$ $\qquad 7 + b = -8$ $\qquad \boxed{b = -15}$

10.- $f(x) = x^2 + bx + c$ / $f'(x) = 2x + b$ / $f''(x) = 2$

- Tg a $y = x$ en $(1, 1)$

$m = 1 \Rightarrow f'(1) = 2 + b = 1 \Rightarrow \boxed{b = -1}$

$f(1) = 1$

$1 + b + c = 1$

$b + c = 0$

$\boxed{b = -1} \longrightarrow \boxed{c = 1} \Longrightarrow \boxed{f(x) = x^2 - x + 1}$

Ec. de la recta tangente a $x = 2$

$f'(x) = 2x - 1 \longrightarrow m = f'(2) = 3$ $\qquad f(2) = 3$

$y - 3 = 3(x - 2)$

$$\boxed{y = 3x - 3}$$